**MCC. 3.OA.5 (Multiplication and Division Properties)**

Apply properties of operations as strategies to multiply and divide.

Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)

Use arrays, area models, and manipulatives to develop understanding of properties.

**ENDURING UNDERSTANDINGS**

Multiplication and division are inverses; they undo each other.

• Multiplication and division can be modeled with arrays.

• Multiplication is commutative, but division is not.

• There are two common situations where division may be used.

o Partition (or fair-sharing) - given the total amount and the number of equal groups, determine how many/much in each group

o Measurement (or repeated subtraction) - given the total amount and the amount in a group, determine how many groups of the same size can be created.

As the divisor increases, the quotient decreases; as the divisor decreases, the quotient increases.

• There is a relationship between the divisor, the dividend, the quotient, and any remainder.

Multiplication facts can be deduced from patterns.

• The associative property of multiplication can be used to simplify computation.

• The distributive property of multiplication allows us to find partial products and then find their sum.

• Patterns are evident when multiplying a number by ten or a multiple of ten.

**ESSENTIAL QUESTIONS**

How are multiplication and division alike and different?

• How are multiplication and division related?

How can I model multiplication by ten?

• How can multiplication and division be used to solve real world problems?

• How can multiplication be represented?

• How can multiplication products be displayed on a multiplication chart?

• How can the same array represent both multiplication and division?

• How can we determine numbers that are missing on a times table chart by knowing multiplication patterns?

How can we divide larger numbers?

• How can we model division?

How can we practice multiplication facts in a meaningful way that will help us remember them?

• How can we use arrays to help develop an understanding of the commutative property?

• How can we use patterns to solve problems?

How can you interpret the product by making equal groups?

• How can you use multiplication facts to solve unknown factor problems?

How does drawing an array help us think about different ways to decompose a number?

How is division an unknown factor problem?

What strategy did you find most efficient when dividing?

What strategies can be used to solve real world division problems?

In what ways can division be represented?

• How can you use what you know about multiplication to help you write your own multiplication problem?

• How can you write a mathematical sentence to represent a multiplication model we have made?

• How do estimation, multiplication, and division help us solve problems in everyday life?

How does an array represent the meaning of multiplication?

How does the order of the digits in a multiplication problem affect the product?

• How does understanding the distributive property help us multiply large numbers?

How is multiplication and division used to solve a problem?

• How is multiplying by ten related to place value?

• How is place value related to multiples of ten?

• How is the associative property of multiplication used in solving a problem?

• How is the commutative property of multiplication evident in an array model?

What are different ways to multiply two factors to get the same product?

• What are strategies for leaning multiplication facts?

• What are the strategies for learning multiplication?

• What happens to a number when it is multiplied by ten?

• What is the relationship between the factors and the product?

• What patterns of multiplication can we discover by studying a times table chart?

• What strategies can be used to find factors or products?

• What strategies can be used to solve multiplication problems?

What strategies can help you solve real world multiplication problems?

**CONCEPTS/SKILLS TO MAINTAIN**

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

● Addition, Subtraction, Multiplication, Division

● Skip counting

● Relationship between addition and multiplication

● Two-dimensional plane figures

● Understanding of arrays

● Solving one-step word problems

● Factors of products

● Commutative Property of Multiplication

● Distributive Property of Multiplication

**STRATEGIES FOR TEACHING AND LEARNING**

**Represent and solve problems involving multiplication and division.**

In Grade 2, students found the total number of objects using rectangular arrays, such as a 5 x 5, and wrote equations to represent the sum. This strategy is a foundation for multiplication because students should make a connection between repeated addition and multiplication.

Students need to experience problem-solving involving equal groups (whole unknown or size of group is unknown) and multiplicative comparison (unknown product, group size unknown or number of groups unknown) as shown in the table in the unit overview. No attempt should be made to teach the abstract structure of these problems.

Encourage students to solve these problems in different ways to show the same idea and be able to explain their thinking verbally and in written expression. Allowing students to present several different strategies provides the opportunity for them to compare strategies.

Sets of counters, number lines to skip count and relate to multiplication and arrays/area models will aid students in solving problems involving multiplication and division. Allow students to model problems using these tools. They should represent the model used as a drawing or equation to find the solution.



This shows multiplication using grouping with 3 groups of 5 objects and can be written as 3 × 5.

Provide a variety of contexts and tasks so that students will have more opportunity to develop and use thinking strategies to support and reinforce learning of basic multiplication and division facts.

Have students create multiplication problem situations in which they interpret the product of whole numbers as the total number of objects in a group and write as an expression. Also, have students create division-problem situations in which they interpret the quotient of whole numbers as the number of shares.

Students can use known multiplication facts to determine the unknown fact in a multiplication or division problem. Have them write a multiplication or division equation and the related multiplication or division equation. For example, to determine the unknown whole number in 27 ÷ = 3, student 27. They should ask themselves questions such as, “How many 3s are in 27 ?” or “3 times what number is 27?” Have them justify their thinking with models or drawings.

Students need to apply properties of operations (commutative, associative and distributive) as strategies to multiply and divide. Applying the concept involved is more important than students knowing the name of the property. Understanding the commutative property of multiplication is developed through the use of models as basic multiplication facts are learned. For example, the result of multiplying 3 x 5 (15) is the same as the result of multiplying 5 x 3 (15).

To find the product of three numbers, students can use what they know about the product of two of the factors and multiply this by the third factor. For example, to multiply 5 x 7 x 2, students know that 5 x 2 is 10. Then, they can use mental math to find the product of 10 x 7 (70). Allow students to use their own strategies and share with the class when applying the associative property of multiplication.

Splitting arrays can help students understand the distributive property. They can use a known fact to learn other facts that may cause difficulty. For example, students can split a 6 x 9 array into 6 groups of 5 and 6 groups of 4; then, add the sums of the groups.



The 6 groups of 5 is 30 and the 6 groups of 4 is 24. Students can write 6 x 9 as 6 x 5 + 6 x 4.

Students’ understanding of the part/whole relationships is critical in understanding the connection between multiplication and division.

**Multiply and divide within 100**

Students need to understand the part/whole relationships in order to understand the connection between multiplication and division. They need to develop efficient strategies that lead to the big ideas of multiplication and division. These big ideas include understanding the properties of operations, such as the commutative and associative properties of multiplication and the distributive property. The naming of the property is not necessary at this stage of learning.

In Grade 2, students found the total number of objects using rectangular arrays, such as a 5 x 5, and wrote equations to represent the sum. This is called unitizing, and it requires students to count groups, not just objects. They see the whole as a number of groups of a number of objects. This strategy is a foundation for multiplication in that students should make a connection between repeated addition and multiplication.

As students create arrays for multiplication using objects or drawing on graph paper, they may discover that three groups of four and four groups of three yield the same results. They should observe that the arrays stay the same, although how they are viewed changes. Provide numerous situations for students to develop this understanding.



To develop an understanding of the distributive property, students need decompose the whole into groups. Arrays can be used to develop this understanding. To find the product of 3 × 9, students can decompose 9 into the sum of 4 and 5 and find 3 × (4 + 5).



The distributive property is the basis for the standard multiplication algorithm that students can use to fluently multiply multi-digit whole numbers in Grade 5.

Once students have an understanding of multiplication using efficient strategies, they should make the connection to division. Using various strategies to solve different contextual problems that use the same two one-digit whole numbers requiring multiplication allows for students to commit to memory all products of two one-digit numbers.

**EVIDENCE OF LEARNING**

By the conclusion of this unit, students should be able to demonstrate the following competencies:

use mental math to multiply and divide

• use estimation to determine reasonableness of products and quotients computed

• be able to read, interpret, solve, and compose simple word problems dealing with multiplication and division

• understand how to use inverse operations to verify accuracy of computation

fluently multiply and divide within 100, using strategies such as the patterns and relationships between multiplication and division

• apply properties of operations (commutative, associative, and distributive) as strategies to multiply and divide

understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

**LEARNZILLION:**

<https://learnzillion.com/lessonsets/226-apply-properties-of-operations-as-strategies-to-multiply-and-divide>